Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Specimen Paper

(Time: 1 hour 30 minutes) | Paper Reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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Pearson

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1. Use the binomial theorem to expand

$$\sqrt[3]{(8-3x)^2} \qquad |x| < \frac{8}{3}$$

in ascending powers of x, up to and including the term in x^3, fully simplifying each term.

(5)

$$\sqrt[3]{(8-3x)^2} = (8-3x)^{\frac{2}{3}}$$

Get the formula for the Binomial Expansion from the formula booklet:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \text{, Range of validity } |x|<1$$

We see that the expansion is in form $(a+bx)^n$, we need to manipulate to get $(1+ax)^n$.

$$(8-3x)^{\frac{2}{3}} = (8)^{\frac{2}{3}}\left(1-\frac{3}{8}x\right)^{\frac{2}{3}}$$

$$= 4\left(1-\frac{3}{8}x\right)^{\frac{2}{3}} \quad \text{B1}$$

Substitute into the Formula:

$$4\left(1-\frac{3}{8}x\right)^{\frac{2}{3}} = 4\left(1 + \frac{2}{3}\left(-\frac{3}{8}x\right) + \frac{\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)}{2!}\left(-\frac{3}{8}x\right)^2 + \frac{\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{3!}\left(-\frac{3}{8}x\right)^3 + \ldots\right)$$

$$= 4\left(1 - \frac{\cancel{6}^{1}}{\cancel{24}_{4}}x + \left(-\frac{1}{9}\right)\left(\frac{\cancel{9}^{1}}{64}x^2\right) + \left(\frac{\cancel{4}^{1}}{\cancel{81}_{3}}\right)\left(-\frac{\cancel{27}^{1}}{\cancel{512}_{128}}x^3\right) + \ldots\right) \quad \text{M1A1}$$

$$= 4 - x - \frac{4}{64}x^2 - \frac{4}{384}x^3 + \ldots$$

$$= 4 - x - \frac{1}{16}x^2 - \frac{1}{96}x^3 + \ldots \quad \text{A1A1}$$

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Question 1 continued

Q1

(Total for Question 1 is 5 marks)

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2. (a) Express $\dfrac{1}{(2x+3)(3x+2)}$ in partial fractions.
(3)

(b) Hence find, in the form $y = f(x)$, the general solution of the differential equation

$$(2x+3)(3x+2)\frac{dy}{dx} = 5y \qquad x > 0,\ y > 0$$
(5)

(a) We are asked to do Partial Fractions

$$\frac{1}{(2x+3)(3x+2)} = \underbrace{\frac{A}{2x+3} + \frac{B}{3x+2}}$$

Manipulate this to get common denominator

$$\frac{1}{(2x+3)(3x+2)} = \frac{A(3x+2)+B(2x+3)}{(2x+3)(3x+2)}$$

Equate the numerators and substitute in values in order to get A, B and C.

$1 = A(3x+2)+B(2x+3)$ B1

choose values to substitute in so that brackets become 0, to make your life easier!

$x = -\frac{2}{3} \Rightarrow 1 = A(\underbrace{3x-\tfrac{2}{3}+2}_{0}) + B(2x-\tfrac{2}{3}+3)$

$1 = \frac{5}{3}B \Rightarrow B = \frac{3}{5}$

$x = -\frac{3}{2} \Rightarrow 1 = A(3x-\tfrac{3}{2}+2) + B(\underbrace{2x-\tfrac{3}{2}+3}_{0})$

$1 = -\frac{5}{2}A \Rightarrow A = -\frac{2}{5}$ M1

$\therefore \dfrac{1}{(2x+3)(3x+2)} = -\dfrac{2}{5(2x+3)} + \dfrac{3}{5(3x+2)}$ A1

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Question 2 continued

(b) $(2x+3)(3x+2)\frac{dy}{dx} = 5y$

$\int \frac{1}{5y}\,dy = \int \frac{1}{(2x+3)(3x+2)}\,dx$ Separate Variables by grouping the like terms on one side

$\int \frac{1}{5} \times \frac{1}{y}\,dy = \int -\frac{2}{5(2x+3)} + \frac{3}{5(3x+2)}\,dx$ use the P.F we got in (a)

B1

$\frac{1}{5}\int \frac{1}{y}\,dy = \int -\frac{2}{5(2x+3)} + \frac{3}{5(3x+2)}\,dx$ take out the constant

$\frac{1}{5}\ln y = -\frac{2}{5}\ln(2x+3) \times \frac{1}{2} + \frac{3}{5}\ln(3x+2) \times \frac{1}{3}$ Integrate

Reverse chain Rule: divide by derivative of the bracket

$\frac{1}{5}\ln y = -\frac{1}{5}\ln(2x+3) + \frac{1}{5}\ln(3x+2) + c$ multiply both sides by 5

M1

let $c = \ln k$

$\ln y = \ln(3x+2) - \ln(2x+3) + \ln k$ apply ln law $\ln a - \ln b = \ln\frac{a}{b}$

A1

$\ln y = \ln\left(\frac{3x+2}{2x+3}\right) + \ln k$ apply ln law $\ln a + \ln b = \ln a \times b$

M1 $\ln y = \ln\left(k\frac{3x+2}{2x+3}\right)$

$e^{\ln y} = e^{\ln\left(k\frac{3x+2}{2x+3}\right)}$ apply $e^{\ln x} = x$

$y = k\left(\frac{3x+2}{2x+3}\right)$

hence solved

A1

Q2

(Total for Question 2 is 8 marks)

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3. Using the substitution $u = 1 + \tan x$, find the exact value of

$$\int_0^{\frac{\pi}{3}} \frac{1}{\cos^2 x + \sin x \cos x}\,dx$$

(6)

To use the given substitution $u = 1 + \tan x$, we need to get new limits, $dx = \ldots$:

New limits:

x		u
$\frac{\pi}{3}$	$u = 1 + \tan(\frac{\pi}{3}) \rightarrow$	$1+\sqrt{3}$
0	$u = 1 + \tan(0) \rightarrow$	1

Differentiate $u = \ldots$ to get dx:

$$u = 1 + \tan x$$

$$\frac{du}{dx} = \sec^2 x$$

$$\Rightarrow dx = \frac{1}{\sec^2 x}\,du$$

B1

Substitute into the Integration:

$$\int_0^{\frac{\pi}{3}} \frac{1}{\cos^2 x + \sin x \cos x}\,dx$$

$$= \int_1^{1+\sqrt{3}} \frac{1}{\cos^2 x + \sin x \cos x} \times \frac{1}{\sec^2 x}\,du$$

$\sec^2 x = \frac{1}{\cos^2 x}$

$$= \int_1^{1+\sqrt{3}} \frac{1}{\frac{1}{\cos^2 x}(\cos^2 x + \sin x \cos x)}\,du$$

$$= \int_1^{1+\sqrt{3}} \frac{1}{1 + \frac{\sin x}{\cos x}}\,du$$

apply $\frac{\sin x}{\cos x} = \tan x$

$$= \int_1^{1+\sqrt{3}} \frac{1}{1 + \tan x}\,du$$

$u = 1 + \tan x$

$$= \int_1^{1+\sqrt{3}} \frac{1}{u}\,du$$

Integrate this

M1A1

$$= \left[\ln u\right]_1^{1+\sqrt{3}}$$

M1M1

$$= \ln(1+\sqrt{3}) - \cancel{\ln(1)}^{0}$$

Substitute in limits

$$= \ln(1+\sqrt{3})$$

A1

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Question 3 continued

Q3

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4.

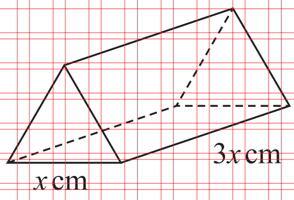


Figure 1

Figure 1 shows a right triangular prism. The cross-section is an equilateral triangle with side x cm and the length of the prism is $3x$ cm, with $x > 2$

Given that the cross-sectional area of the prism is changing at a rate of $(2 - x)\ \text{cm}^2\,\text{s}^{-1}$

(a) find, in terms of x, an expression for $\dfrac{dx}{dt}$

(4)

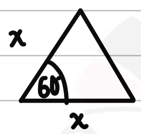
(b) find the rate of decrease of the volume of the prism when $x = 2.05$

(4)

(a) We are given that $\dfrac{dA}{dt} = (2-x)\,\text{cm}^2\text{s}^{-1}$ where A is cross section area.

B1

The cross sectional area is an equilateral triangle with side x:



Hence using $A = \frac{1}{2}ab\sin c$ we can get the area:

$$A = \frac{1}{2}x^2\sin 60$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2}x^2$$

$$\Rightarrow A = \frac{\sqrt{3}}{4}x^2$$

B1

Differentiate to get $\dfrac{dA}{dx}$:

$$\frac{dA}{dx} = \frac{\sqrt{3}}{2}x$$

We are asked to find $\dfrac{dx}{dx}$:

M1

given

$$\frac{dx}{dt} = \frac{dA}{dt} \times \frac{dx}{dA}$$

reciprocal of what we found above

$$\therefore \frac{dx}{dt} = (2-x) \times \frac{2}{x\sqrt{3}}$$

$$\Rightarrow \frac{dx}{dt} = \frac{4-2x}{x\sqrt{3}}$$

found

A1

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Question 4 continued

(b) We're asked to find the rate of decrease of volume $\therefore \frac{dV}{dt}$.

Volume of the prism:

V = cross section × length

A found in (a)

Hence: $V = \left(\frac{\sqrt{3}}{4}x^2\right) \times 3x$ given

$$V = \frac{3\sqrt{3}}{4}x^3$$

Differentiate to get $\frac{dV}{dx}$:

$$\frac{dV}{dx} = \frac{9\sqrt{3}}{4}x^2 \quad \text{M1A1}$$

given, $\frac{dx}{dt} = (2-x)$

To get $\frac{dV}{dt}$: $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$

$$\Rightarrow \frac{dV}{dt} = \left(\frac{9\sqrt{3}}{4}x^2\right)(2-x)$$

$$\left.\frac{dV}{dt}\right|_{x=2.05} = \frac{9\sqrt{3}}{4}(2.05)^2(2-2.05) = -\frac{369}{800}$$

plug this into your calculator

M1

Hence Rate of Decrease is $\frac{369}{800}$ cm^3 s^{-1} A1

Q4

(Total for Question 4 is 8 marks)

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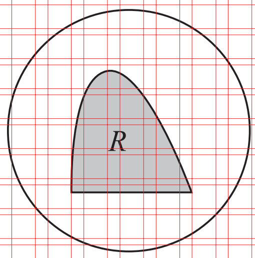

Figure 2

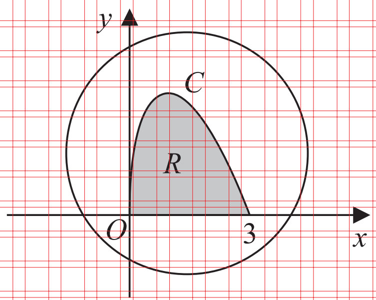

Figure 3

Figure 2 shows the design for a logo of a fishing club. The design is made up of a shaded region R inside a circle of radius 3

The region R is the area between the x-axis and the curve C shown in Figure 3.

The curve C is defined by the parametric equations

$$x = 4\sin^2 t \qquad y = 3\sin 3t \qquad 0 \leqslant t \leqslant \frac{\pi}{3}$$

(a) Prove that

$$\sin 3t \equiv 3\sin t - 4\sin^3 t$$

(3)

(b) Using algebraic integration find, to 3 significant figures, the percentage of the circle that is shaded as part of the logo.

(8)

(a) $LHS = \sin 3t$

$= \sin(2t + t)$ apply addition formula: $\sin(a+b) = \sin a\cos b + \cos a\sin b$

M1 $= \sin 2t\cos t + \cos 2t\sin t$

double angle formula: $\sin 2A = 2\sin A\cos A$ $= (2\sin t\cos t)\cos t + (1 - 2\sin^2 t)\sin t$ M1

double angle formula: $\cos 2A = 1 - 2\sin^2 t$

$= 2\sin t\cos^2 t + \sin t - 2\sin^3 t$

$\cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$

$= 2\sin t(1 - \sin^2 t) + \sin t - 2\sin^3 t$

$= 2\sin t - 2\sin^3 t + \sin t - 2\sin^3 t$

$= 3\sin t - 4\sin^3 t = RHS$ hence shown

A1

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Question 5 continued

(b) Let's first get area of circle:

$A = \pi r^2 \rightarrow A = \pi(3)^2 \therefore A = 9\pi$

★ Parametric Integration

area $= \int_a^b y \frac{dx}{dt} dt$

Hence we need $\frac{dx}{dt}$:

$x = 4\sin^2 t$

$\frac{dx}{dt} = 8\sin t\cos t$ chain rule: multiply by derivative of the bracket

Substitute into the Integration: B1

$R = \int_0^{\frac{\pi}{3}} \overbrace{3\sin 3t}^{y=\dots} \times 8\sin t\cos t \, dt$ M1

$= \int_0^{\frac{\pi}{3}} 24\sin 3t \sin t\cos t \, dt$

result proven in (a)

$= 24\int_0^{\frac{\pi}{3}} (3\sin t - 4\sin^3 t)\sin t\cos t \, dt$

$= 24\int_0^{\frac{\pi}{3}} 3\sin^2 t\cos t - 4\sin^4 t\cos t \, dt$ A1

Integration by Recognition, Form:

$\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$

$= 24\left[\frac{3\sin^3 t}{3} - \frac{4\sin^5 t}{5}\right]_0^{\frac{\pi}{3}}$ M1A1

$= 24\left(\left(\sin^3\left(\frac{\pi}{3}\right) - \frac{4}{5}\sin^5\left(\frac{\pi}{3}\right)\right) - (0 - 0)\right)$

$= 24\left(\left(\frac{\sqrt{3}}{2}\right)^3 - \frac{4}{5}\left(\frac{\sqrt{3}}{2}\right)^5\right)$

$= 24\left(\frac{3\sqrt{3}}{8} - \frac{\cancel{4}^{1}}{5}\left(\frac{9\sqrt{3}}{\cancel{32}_{8}}\right)\right)$

$= 24\left(\frac{3\sqrt{3}}{8} - \frac{9\sqrt{3}}{40}\right) = 24\left(\frac{6\sqrt{3}}{40}\right)$

$= \frac{18\sqrt{3}}{5}$ dM1

$\therefore$ Proportion Shaded $= \frac{\frac{18\sqrt{3}}{5}}{9\pi} \times 100$ M1

$= 22.1\%$ A1

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Question 5 continued

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Question 5 continued

Q5

(Total for Question 5 is 11 marks)

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6. Relative to a fixed origin O, the points A and B have position vectors $\overrightarrow{OA} = \begin{pmatrix} 9 \\ 5 \\ 12 \end{pmatrix}$

and $\overrightarrow{OB} = \begin{pmatrix} 13 \\ -3 \\ 4 \end{pmatrix}$ respectively.

The line l_1 has equation

$$\mathbf{r} = \overrightarrow{OA} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

and the line l_2 has equation

$$\mathbf{r} = \overrightarrow{OB} + \mu \begin{pmatrix} 10 \\ -5 \\ 1 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 intersect at a point X, and find the coordinates of X.
(6)

(b) Find the coordinates of the point P such that $XAPB$ is a parallelogram.
(2)

(c) Show that $XAPB$ is a rhombus.
(2)

(d) Prove by contradiction that $XAPB$ is not a square.
(3)

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Question 6 continued

(a) Equate the general points of the lines to get the values of λ and μ where they intersect:

$$\begin{pmatrix} 9+2\lambda \\ 5+\lambda \\ 12+3\lambda \end{pmatrix} = \begin{pmatrix} 13+10\mu \\ -3-5\mu \\ 4+\mu \end{pmatrix}$$

Solve simultaneously x and y components:

$$9+2\lambda = 13+10\mu$$ solve by elimination

$5+\lambda=-3-5\mu \; (\times 2)$ $\quad 10+2\lambda = -6-10\mu$ $+$ M1

$$19+4\lambda = 7$$

$$4\lambda = -12$$

$$\lambda = -3 \rightarrow 9+2(-3) = 13+10\mu$$

$$\mu = -1$$ M1A1

Check that $\lambda=-3, \mu=-1$ satisfies z component:

$$12+3(-3) = 4+(-1)$$

$$12-9 = 4-1$$

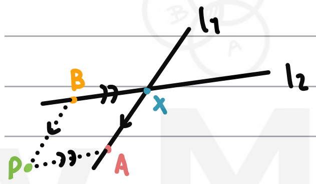
$$3=3 \checkmark$$ B1

Hence they intersect at point x:

$$\vec{OX} = \begin{pmatrix} 9 \\ 5 \\ 12 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$ (using l_1)

$$\vec{OX} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$$ M1A1

(b) Diagram



For it to be a parallelogram, $\vec{AP}$ must be parallel to $\vec{XB}$ and $\vec{XA}$ parallel to $\vec{BP}$, and $|\vec{AP}| = |\vec{XB}|$ as well as $|\vec{XA}| = |\vec{BP}|$.

Hence we can get $\vec{OP}$ like this:

$$\vec{OP} = \vec{OA} + \vec{XB}$$

$$= \vec{OA} + (\vec{OB} - \vec{OX})$$

$$= \begin{pmatrix} 9 \\ 5 \\ 12 \end{pmatrix} + \left(\begin{pmatrix} 13 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}\right)$$ M1

$$= \begin{pmatrix} 9 \\ 5 \\ 12 \end{pmatrix} + \begin{pmatrix} 10 \\ -5 \\ 1 \end{pmatrix}$$ $\vec{XB}$

$$\vec{OP} = \begin{pmatrix} 19 \\ 0 \\ 13 \end{pmatrix}$$ A1

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Question 6 continued

(c) For it to be a rhombus, $\vec{XA}$ and $\vec{XB}$ must be the same magnitude.

part(b) $\vec{XB} = \begin{pmatrix}10\\-5\\1\end{pmatrix}$, $\vec{XA} = \vec{OA} - \vec{OX}$ part (a)

$= \begin{pmatrix}9\\5\\12\end{pmatrix} - \begin{pmatrix}3\\2\\3\end{pmatrix} = \begin{pmatrix}6\\3\\9\end{pmatrix} = \vec{XA}$

Formula for Magnitude of a vector:

Let $\vec{AB} = \begin{pmatrix}a\\b\\c\end{pmatrix}$.

$|\vec{AB}| = \sqrt{a^2 + b^2 + c^2}$

$|\vec{XA}| = \sqrt{6^2 + 3^2 + 9^2} = \sqrt{126}$

$|\vec{XB}| = \sqrt{10^2 + (-5)^2 + 1^2} = \sqrt{126}$ M1

Hence the sides are equal and it is a rhombus.

A1

(d) Make an Assumption:

"Assume it's a square, hence the sides are perpendicular"

M1

We want our assumption to be the opposite of what we are trying to prove. In the Proof, we are trying to contradict the assumption

Use Dot Product to show that $\vec{XA}$ and $\vec{XB}$ are perpendicular

Formula for dot product:

$\begin{pmatrix}a\\b\\c\end{pmatrix} \cdot \begin{pmatrix}d\\e\\f\end{pmatrix} = ad + be + cf$

$\begin{pmatrix}6\\3\\9\end{pmatrix} \cdot \begin{pmatrix}10\\-5\\1\end{pmatrix} = 60 - 15 + 9 = 54$

Since dot product $\neq 0$, the sides are not perpendicular. This is a contradiction, hence XAPB is not a square. A1

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Question 6 continued

Q6

(Total for Question 6 is 13 marks)

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7. (a) Use integration by parts to show that

$$\int e^{2x} \cos 2x \, dx = \frac{e^{2x}(\sin 2x + \cos 2x)}{4} + c$$

where c is an arbitrary constant.

(5)

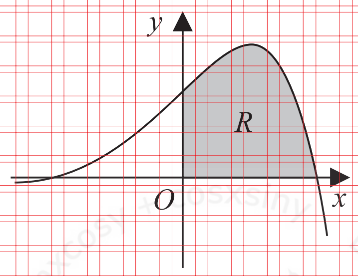

Figure 4

Figure 4 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = e^x \cos x$$

The finite region R, shown shaded in Figure 4, is bounded by the curve, the positive y-axis and the positive x-axis.

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Using the result from part (a), find the exact volume of the solid formed.

(6)

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Question 7 continued

(a) Integration by Parts:

$\int uv'dx = uv - \int u'v\,dx$

$$\int e^{2x}\cos 2x\,dx$$

$u = e^{2x} \xrightarrow{\frac{d}{dx}} u' = 2e^{2x}$

$v = \frac{1}{2}\sin 2x \xleftarrow{\int} v' = \cos 2x$

$= \frac{1}{2}e^{2x}\sin 2x - \int e^{2x}\sin 2x\,dx$ M1

By parts again

$u = e^{2x} \xrightarrow{\frac{d}{dx}} u' = 2e^{2x}$

$v = -\frac{1}{2}\cos 2x \xleftarrow{\int} v' = \sin 2x$

$= \frac{1}{2}e^{2x}\sin 2x - \left(-\frac{1}{2}e^{2x}\cos 2x - \int -e^{2x}\cos 2x\,dx\right)$ M1A1

Reintroduce our initial Integral on the LHS and use it to get rid of the infinite "By Parts".

$\int e^{2x}\cos 2x\,dx = \frac{1}{2}e^{2x}\sin 2x + \frac{1}{2}e^{2x}\cos 2x - \int e^{2x}\cos 2x\,dx$

$2\int e^{2x}\cos 2x\,dx = \frac{1}{2}e^{2x}\sin 2x + \frac{1}{2}e^{2x}\cos 2x$ M1

$\int e^{2x}\cos 2x\,dx = \frac{1}{4}e^{2x}\sin 2x + \frac{1}{4}e^{2x}\cos 2x$ divide both sides by 2

$\therefore \int e^{2x}\cos 2x\,dx = \frac{1}{4}e^{2x}(\sin 2x + \cos 2x) + c$ A1

hence shown

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Question 7 continued

(b) Formula for Volume of Revolution around the x-axis:

$$V = \pi\int_a^b y^2\,dx$$

where a and b are the points on the x-axis bounding the area

Get the upper limit:

$$0 = e^x\cos x$$
$$\cos x = 0$$
$$x = \frac{\pi}{2}$$

Substitute in:

$$V = \pi\int_0^{\frac{\pi}{2}} (e^x\cos x)^2\,dx \quad \text{B1}$$

$$= \pi\int_0^{\frac{\pi}{2}} e^{2x}\cos^2 x\,dx$$

double angle formula: $\cos 2A = 2\cos^2 A - 1$

$\cos^2 A = \frac{1}{2}(\cos 2A + 1)$

$$= \pi\int_0^{\frac{\pi}{2}} e^{2x}\left(\tfrac{1}{2}(\cos 2x + 1)\right)dx \quad \text{M1}$$

$$= \pi\int_0^{\frac{\pi}{2}} \frac{1}{2}e^{2x}(\cos 2x + 1)\,dx$$

$$= \frac{1}{2}\pi\int_0^{\frac{\pi}{2}} e^{2x}\cos 2x + e^{2x}\,dx$$

take out $\frac{1}{2}$ and expand

$$= \frac{1}{2}\pi\int_0^{\frac{\pi}{2}} e^{2x}\,dx + \frac{1}{2}\pi\int_0^{\frac{\pi}{2}} e^{2x}\cos 2x\,dx$$

split the Integral

we got this in (a)

$$= \left[\frac{\pi}{2}\cdot\frac{e^{2x}}{2} + \frac{\pi}{2}\left(\left(\frac{1}{4}e^{2x}(\sin 2x + \cos 2x)\right)\right)\right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi e^{2x}}{4} + \frac{\pi}{8}e^{2x}(\sin 2x + \cos 2x)\right]_0^{\frac{\pi}{2}} \quad \text{M1A1}$$

$$= \left[\frac{\pi}{8}e^{2x}\left(2 + \sin 2x + \cos 2x\right)\right]_0^{\frac{\pi}{2}}$$

factor out $\frac{\pi}{8}e^{2x}$

$$= \frac{\pi}{8}e^{\pi}(2 + \sin\pi + \cos\pi) - \frac{\pi}{8}e^0(2 + \sin 0 + \cos 0)$$

substitute in limits

M1

$$= \frac{\pi}{8}e^{\pi} - \frac{3\pi}{8}$$

$$\text{Volume} = \frac{\pi}{8}(e^{\pi} - 3) \quad \text{A1}$$

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Question 7 continued

Q7

(Total for Question 7 is 11 marks)

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8. The curve C has equation

$$11e^{x-2y} + 5xy^2 = 1 + 5x^2$$

The point $P(2, 1)$ lies on C.

(a) Show that at P, $\frac{dy}{dx} = -2$

(5)

(b) Find the exact coordinates of the points where the normal to C at P crosses the curve C again.

(8)

(a)

★ Implicit differentiation:

When differentiating y, multiply by $\frac{dy}{dx}$

$$11e^{x-2y} + 5xy^2 = 1 + 5x^2$$

Product Rule: $\frac{d}{dx}(uv) = uv' + u'v$

$$11e^{x-2y} \times \left(1 - 2\frac{dy}{dx}\right) + 5y^2 + 5x\left(2y\frac{dy}{dx}\right) = 10x$$

chain rule: multiply by the derivative of the bracket

M1A1B1

$$11\left(1 - 2\frac{dy}{dx}\right)e^{x-2y} + 5y^2 + 10xy\frac{dy}{dx} = 10x$$

Substitute P(2,1) and Solve for $\frac{dy}{dx}$:

$$11\left(1 - 2\frac{dy}{dx}\right)e^{2-2(1)} + 5(1)^2 + 10(1)(2)\frac{dy}{dx} = 10(2)$$

$$\left(11 - 22\frac{dy}{dx}\right)\cancel{e^0}^{\,1} + 5 + 20\frac{dy}{dx} = 20$$

$$16 - 22\frac{dy}{dx} + 20\frac{dy}{dx} = 20$$

$$-2\frac{dy}{dx} = 4$$ isolate $\frac{dy}{dx}$ on one side

$$\therefore \frac{dy}{dx} = -2$$ Hence shown

dM1A1

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Question 8 continued

(b) We are given $\frac{dy}{dx} = -2$ at P, which is the gradient of the tangent.

Get the gradient of the normal:

$m(\text{normal}) = -\frac{1}{m(\text{tangent})} \Rightarrow m(\text{norm}) = -\frac{1}{-2} \quad \therefore \quad m = \frac{1}{2}$ M1

Use $y - y_1 = m(x - x_1)$ to get the equation of the line:

$m = \frac{1}{2}$, $P(2,1) \Rightarrow$

$$y - 1 = \frac{1}{2}(x - 2) \quad \text{A1}$$
$$2y - 2 = x - 2$$
$$x = 2y$$

Substitute this back into the given initial equation to get the points of intersection:

$$11e^{\cancel{2y - 2y}^{\,0}} + 5(2y)y^2 = 1 + 5(2y)^2 \quad \text{M1}$$
$$11 + 10y^3 = 1 + 20y^2$$
$$\text{A1} \quad 10y^3 - 20y^2 + 10 = 0 \quad \text{divide both sides by 10}$$
$$y^3 - 2y^2 + 1 = 0$$

Factorize – it's easy since we know $(y-1)$ is a factor (from given point $P(2,1)$ being on curve c and the normal)

$$(y - 1)(y^2 - y - 1) = 0 \quad \text{M1}$$

$y = 1$

Use Quadratic Formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$y = \frac{1 \pm \sqrt{1^2 - 4(1)(-1)}}{2} \quad \text{M1}$$

$$y = \frac{1 \pm \sqrt{5}}{2}$$

Use this and $x = 2y$ to get complete coordinates.

The curves meet again at:

$\left(1 + \sqrt{5}, \frac{1 + \sqrt{5}}{2}\right)$ and $\left(1 - \sqrt{5}, \frac{1 - \sqrt{5}}{2}\right)$ A1A1

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Question 8 continued

Q8

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

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